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# **COAL BLENDING OPTIMIZATION UNDER UNCERTAINTY**

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#### **ABSTRACT**

Coal blending is one of several options available for reducing sulfur emissions from coal-fired power plants. However, decisions about coal blending must deal with uncertainty and variability in coal properties, and with the effect of off-design coal characteristics on power plant performance and cost. To deal with these issues, a multi-objective chance-constrained optimization model is developed for an illustrative coal blending problem. Sulfur content, ash content and heating value are considered as normally distributed random variables. The objectives of the model include minimizing: (1) the expected (mean) costs of coal blending; and  $(2)$  the variance of coal blending costs. The cost objective function includes coal purchasing cost, ash disposal cost, sulfur removal cost, and fuel switching costs. Chance constraints include several risk measures, such as the probability of exceeding the sulfur emission standard. The model is solved using a mixed integer non-linear optimizer. Several results are presented to illustrate the model. Directions for future work are described.

## **INTRODUCTION**

The 1990 Clean Air Act Amendment regulations on sulfur emissions from coal-fired power plants have motivated a search for cost-effective compliance options, particularly for plants burning high sulfur coal without any sulfur emissions control technology. Because the capital and operating costs of post-combustion flue gas desulfurization (FGD) systems are potentially high, an attractive alternative may be switching from high sulfur to low sulfur coal, or blending coals to reduce overall sulfur content. This is a particularly attractive alternative for small or old power plants, where the levelized costs of retrofitting FGD systems would be high. Furthermore, coal switching or blending to meet the Phase I requirements of the Clean Air Act Amendment may enable the postponement of costly scrubber retrofits at some plants, thereby allowing utilities to become familiar with the sulfur emission allowance trading market, observe the actual market prices for emissions allowances, and choose potentially more cost-effective clean coal technologies than scrubbers for the Phase II requirements [1-4].

Economical operation of a power plant requires careful attention to fuel quality, such as sulfur content, ash content and heating value. However, coal properties are inherently variable even within a single coal seam and, from the perspective of a power plant, over time [5]. For example, Cheng and *et al.* [6] characterize coal property variability using time series models applied to two specific coals. Variability in coal properties poses yet another challenge for power plant operation, due to the effect of coal properties on potentially all major power plant systems. Thus, a key motivation for coal blending is to minimize the variance in coal properties over time, to enable more predictable and economical plant operation with a minimum in equipment setting adjustments [5,7- 9]. Since the properties of coal are random variables, there is a risk that the specified requirement of the mixture can not be met 100 percent of the time. Therefore, one must consider the risk (or, conversely, the reliability) associated with potential exceedance of coal quality specifications.

Understanding this risk, in terms of the multi-dimensional constraints imposed on coal properties, should be of interest to coal suppliers, coal users, and the regulatory community. Given a characterization of variability in the properties of coals comprising a blend, we may ask the following questions:

- (1) How should we specify coal quality in terms of constraints and the probability of exceeding those constraints due to variability in coal properties?
- (2) Given an explicit probabilistic description of acceptable blended coal quality, how can one optimize coal blends to minimize emissions and/or cost?
- (3) How can the variance in coal properties be minimized by coal blending?
- (4) What are the trade-offs between different objectives for coal blending (e.g., minimizing expected cost, expected emissions, or variance in cost or emissions)?
- (5) What is the benefit of reducing coal property measurement error?

This paper will answer questions such as these taking into account the probabilistic nature of coal properties and the need to consider the effects of changes in coal properties on plant performance, emissions, and cost. A mathematical programming model is developed and applied to generate insight into: (1) coal blending practice for a power plant; (2) implications of coal blending for regulatory compliance; and (3) investment decision making for power plant modifications required for coal blending and for improvement of coal measurements.

## **MATHEMATICAL APPROACHES TO COAL BLENDING**

Gershon [10] describes four computerized approaches for evaluating coal blending strategies. They are spreadsheet analysis, computerized search, expert systems and linear programming (LP). Of these, LP is the most rigorous optimization technique. It can consider all coal qualities simultaneously instead of one at a time. Coal blending analysis using LP has been employed by many [e.g., 7,8,11,12]. A key limitation of the LP models are their inability to deal quantitatively with variation in coal properties.

In a real decision making environment, it is often necessary to consider more than one objective. Multiple objectives may be competing and require trade-offs (e.g., emissions and cost). Furthermore, the random nature of coal properties suggests other objectives. Variability in coal properties in individual coals results in variability in power plant operating and maintenance (O&M) costs which are influenced by coal properties (e.g., ash disposal costs). Therefore, an additional objective of coal blending may be to minimize the variance in emissions and/or cost. We also may be interested in minimizing both the expected value and variance of coal blending costs.

Chance-Constrained Programming (CCP) is an optimization approach which can deal explicitly with variability and, hence, overcome the shortcomings of LP approaches as applied to coal blending [13]. CCP incorporates so-called "chance constraints" which include an explicit measure of the reliability (probability) with which the constraints must be met. Through the use of standard probability distribution models which are analytically tractable, it is possible to convert the chance constraints into "deterministic equivalents." This enables CCP models to be implemented using standard mathematical programming packages. CCP techniques have been applied to many environmental system optimization problems [14,15]

# **VARIABILITY IN COAL PROPERTIES**

The properties of coals, such as sulfur content, ash content, and heating value, are treated here as random variables to reflect their variability and measurement uncertainties. Cheng et al. [6] have statistically characterized the properties of selected coals using time-series models. Table 1 summarizes the statistical properties of two coals. These data are for medium sulfur coals. Therefore, in our analyses of coal blending options, we assume that a flue gas desulfurization system with 90 percent sulfur capture is required. The analysis framework, however, can be applied to cases in which coal blending substitutes for FGD as an emission control option.

Table 1. Coal Characteristics and Prices <sup>a</sup>								
		Coal 1	Coal 2					
Description		σ	μ					
Sulfur Content, %	3.22	0.37	2.73	0.28				
Ash Content, %	19.80	2.67	12.09	1.99				
Heating Value, Btu/lb	11,220	2,369 <sup>b</sup>	12,440	1,840 <sup>b</sup>				
Price, \$/ton	30 <sup>c</sup>		40 <sup>c</sup>					

Table 1. Coal Characteristics and Prices<sup>a</sup>

a Data are from Cheng et al. [6] except where noted.  $\mu$  = mean, σ = standard deviation

b Estimated based on data by Cheng et al.

c Based on typical values for medium sulfur coals and an illustrative price premium for a lower sulfur coal.

## **A MULTI-OBJECTIVE CHANCE CONSTRAINT PROGRAMMING MODEL**

The variability of coal properties implies a need to include risk and reliability criteria in any coal blending optimization model. To deal with the probabilistic nature of coal properties, and the need to consider multiple objectives, a multi-objective CCP model is developed. Here, a CCP formulation is employed which includes two objectives: (1) minimize the expected (mean) costs of coal blending; and (2) minimize the variance of coal blending costs. The model formulation easily can be extended to include additional objectives regarding emissions or plant operations**.** Coal blending costs include the purchase price of each coal, differences in ash disposal costs compared to the design coal, differences in FGD sorbent and sludge disposal costs, incremental capital costs required for plant modifications to accommodate off-design coals, and other incremental operating and maintenance (O&M) costs. Because costs are sensitive to coal properties, which are random variables, the costs associated with coal blending are themselves uncertain. Because selected costs are incurred only if more than one coal is used, the cost model is formulated using a mixed integer approach.

The chance constraints to the optimization model include: (1) sulfur emissions must be less than or equal an emission limit with a certain reliability (e.g., 95 percent); (2) the blended coal ash content must conform to a specification with a given reliability; and (3) the average coal heating value input must conform to the plant's electric energy output requirement. The latter constraint is required because variability in heating values and differences in heating values between coals in the blend affect the coal mass flow requirement for a given plant output. The chance constraint for sulfur emissions illustrates the model formulation:

$$
\Pr[(1-\eta)(\sum x_i S_i) \le S^*] \ge \alpha_s \tag{1}
$$

The probability must be greater than a reliability limit,  $\alpha_s$ , that the controlled emissions, taking into account FGD removal efficiency,  $\eta$ , the sulfur content of each coal in the blend,  $S_i$ , and the mass fraction of each coal,  $x_i$ , is less than a specific limit,  $S^*$ . The details of model formulation for the objective function and multi-objective mixed-integer CCP are given by Shih and Frey [16]. The key inputs to the model include the coal properties described in Table 1 and assumptions regarding the constraints of the model. The latter are given in Table 2.

		Constraint		Reliability	
Description	Units	Symbol	Value		Symbol   Probability
<b>Sulfur Emissions</b>	Equiv. wt-% of coal	$S^*$	0.36	$\alpha$ S	0.95
Ash Content,	wt-% of coal	$A^*$	24.0	$\alpha_A$	0.90
Coal Heat Inputa	$106$ BTU/hr	$H^*$	4875		

Table 2. Illustrative Assumptions for Chance Constraints

 $A^*$  H<sup>\*</sup> is the nominal coal heat input required to meet a given electricity generation demand. We assume a 500 MW power plant with a heat rate of 9,750 BTU/kWh.



Figure 1. The Trade-Off Between Expected Value and Standard Deviation of Cost

Table 3. Optimal Coal Blends for Different Weightings of Minimizing Expected Value and Standard Deviation of Cost

Node	Weights $(w_1, w_2)$	<b>Expected Cost</b> $E(C)$ , $\frac{\pi}{2}$	Standard Dev. of cost, \$/hr	Coal Feed Rate <sup>a</sup> tons/hr $(m_1, m_2)$
	(0.00, 1.00)	9694	59	(68.6, 134)
2	(0.01, 0.99)	9526	60	(85, 8, 118)
3	(0.02, 0.98)	9338	63	(105, 101)
4	(0.03, 0.97)	9102	69	(129, 79.4)
5	(0.04, 0.96)	8746	82.3	(166, 46.5)
6	(0.05, 0.95)	8742	82.4	(166, 46.2)

 $a_{m_1}$  = coal feed rate for Coal 1; m<sub>2</sub> = coal feed rate for Coal 2.

#### **ILLUSTRATIVE MODEL APPLICATIONS**

We consider two case studies to illustrate the benefits of a probabilistic approach to the analysis of coal blending options. The first concerns the trade-offs between minimizing the mean and variance of cost. The second focuses on a sensitivity analysis of the sulfur emission constraint and its implications for coal blending requirements and cost.

#### Trade-Off Between Expectation and Variance of Cost

Variance in cost is obtained for any given coal blend due to the stochastic nature of the coal properties, which in turn affect plant performance and cost. The results for making trade-offs between the two objectives of minimizing expected cost and standard deviation (SD) in cost were obtained using the weighting method of Cohon [17]. The objective function may be written as:

$$
\text{Min } Z = w_1 \text{ E(Cost)} + w_2 \text{ SD(Cost)} \tag{2}
$$

Figure 1 shows the effect on the optimal solution of different values of the weights,  $w_1$  and  $w_2$ , on the competing objectives. The numbers on each point of the graph refer to entries in Table 3, which provides details regarding the weights employed in the objective function, the optimal values obtained for expected cost and standard deviation in cost, and the associated optimal blends for the two coals considered here. Point 1 on the graph represents the case in which the standard deviation of cost is minimized, whereas Point 6 represents a combination of minimized expected cost and standard deviation in cost. The results indicate that efforts focused on minimizing the expected value of the cost will increase the variance in the optimal cost, while efforts to minimize variance in cost will result in an increased expected cost. To reduce the variance in cost over the range considered here, the proportion of the cleaner coal, which has a lower variance in properties that affect cost, must be increased. In this case, a proportion of 134 tons/hour of the cleaner coal in the blend yields the minimum variance case. To reduce the expected value of cost, a higher proportion of the dirtier coal must be employed in the blend. However, the sulfur emission chance constraint becomes binding when 166 tons/hour of the blend is comprised of the dirty coal.

Therefore, any further increase in the weight on minimizing expected cost will not change the solution.

In a practical application, a power plant operator may be willing to give up some expected cost savings in order to reduce the variance in process operating conditions, which would reduce the number of changes in equipment settings and provide more stable plant operations.

#### Effect of Sulfur Emissions Reliability Constraint on Minimum Expected Cost

Here we consider an objective of minimizing expected cost and its sensitivity to assumptions regarding the sulfur emissions requirement and the reliability with which the requirement must be met. The results are illustrated in Figure 2, where the minimum expected cost is plotted versus the reliability with which the emission constraint is met for three sulfur emission requirements. These results were obtained using the constraint method of Cohon [17].

Each of the sulfur emission requirement curves in Figure 2 has a step jump. To the left-hand side of the jump, only the single base coal is adopted. At the point of the jump, the cleaner coal is included in the blend to meet the reliability requirement. There is an associated jump in cost due to the plant modifications and the associated capital and O&M costs required to accommodate the new coal type. As the reliability criteria becomes more stringent, a larger proportion of the cleaner coal must be included in the blend, further increasing costs. For any sulfur emission level, there is a point beyond which the reliability criteria become infeasible. For example, given the two available type of coals, and a sulfur emission requirement of no more than 0.35 percent equivalent coal sulfur content, the maximum reliability that can be obtained is 99.8 percent (corresponding to blending 34.4 tons/hr of Coal 1 and 165 tons/hr of Coal 2). To be able to comply with the emission requirement with a higher reliability than this would require inclusion of a third coal in the blend with a lower sulfur content and/or lower variance in sulfur content than the cleaner of the two coals considered here.



Figure 2. Trade-Off Between Expectation of Cost versus Reliability of Meeting the Sulfur Emission Constraint  $(\alpha_S)$ .

## **CONCLUSIONS AND FUTURE RESEARCH**

We have demonstrated the use of the Chance Constrained Programming (CCP) technique for coal blending decision making. This technique permits explicit consideration of the variability inherent in coal characteristics, which are ignored in traditional linear programming approaches to coal blending optimization. Furthermore, the CCP framework enables explicit evaluation of the effects of reliability criteria on coal blending decisions. The case studies described here have illustrated the nature of competing objectives faced by decision makers regarding emissions and cost, both with respect to minimizing expected values and standard deviations. The latter consideration is important from both a plant operations and a regulatory compliance perspective.

As part of ongoing work, we are extending the case studies presented here to include additional objectives. These will include minimizing: (1) the expected value of sulfur emissions; (2) the variance of sulfur emissions; and (3) the tolerance with which the plant meets electrical demand, given variability in coal heating values. The model applications will be extended to include case studies exploring trade-offs among various combinations of objectives, such as minimizing expected cost and the variance in sulfur emissions. Analyses will be performed to illustrate how CCP can be used to help identify and specify specifications for coal quality. Furthermore, the CCP approach will be used to evaluate the potential benefits of more accurate measurements of coal properties.

In the future, would like to extend the application of CCP to coal blends consisting of more than two coals, as well as to deal more explicitly with the real-time implications of coal blending decisions as a function of time series data for coal properties. As part of such work, we envision revisiting the assumptions regarding distribution shapes and dependences. Furthermore, the objective function for coal blending can be tailored to site specific needs, and to consideration of additional attributes such as  $NO<sub>x</sub>$  or  $CO<sub>2</sub>$  emissions.

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